Differential Ordinary Lumpability in Markovian Process Algebra

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Extended Abstract

In Markovian process algebra, fluid semantics interpret a term with a system of coupled ordinary differential equations (ODEs). This has proven useful in approximating the stochastic behaviour of models consisting of groups of many independent replicas of sequential components characterised by small local state spaces. The size of the underlying continuous-time Markov chain (CTMC) is well known to be at worst exponential in the population of such components. Instead, the fluid semantics defines a single ODE for each local state, independently from the multiplicities. The solution to each ODE estimates the time-course evolution of average population of components in that state (e.g., [6, 3, 9]).

We are developing a theory of aggregation for ODE systems induced by a process algebra with fluid semantics [7]. To illustrate and put in context our contribution, let us draw a parallel with established results of aggregation of CTMCs obtained from a Markovian semantics (e.g., [2, 5, 4]). This has involved finding suitable process algebraic behavioural relations that induce a partition of the CTMC state space which satisfies the property of *ordinary lumpability* [8, 1]: a smaller CTMC can be constructed where each state (a *macro-state*) is the representative of the states in a partition block; the probability of being in a macro-state is equal to the sum of the probabilities of being in the block's states.

In our proposal we proceed in an analogous fashion. We introduce *differential ordinary lumpability* (DOL), an equivalence relation over the local states of a process algebra model that captures symmetries in the fluid semantics according to the well-known notion of *exact lumpability* for ODEs [10]. Specifically, given an ODE system and a partition of its ODEs, the system can be rewritten in terms of the variables aggregated according to the partition.

We consider two versions of DOL that trade increased coarsening capability, via less discriminating power, for congruence, which allows compositional applications of aggregations. The first variant is more effective in terms of model reduction because discriminates less behaviour. For instance, it may be able to relate local states that are equal up to a renaming or a collapsing of some action types. While this can yield coarser aggregations, it clearly does not allow for compositional reasoning. The second version of DOL, being a stronger variant of the former, is instead a congruence with respect to the composition operator of our process algebra. In both cases, establishing DOL involves computationally-expensive *semantic checks*, i.e., tests of equalities of symbolic expressions depending on ODE variables. A characterisation of DOL in terms of properties which only require syntactic checks on the process term is thus provided, having the potential to be more efficient to implement on a computer.

As with all analogous results available in the literature for both Markovian and fluid semantics, DOL gives only sufficient conditions for aggregation. Although proving necessity eluded us, we were not able to find examples of aggregations of systems specified in our process algebra which are not characterised

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by DOL (in its weaker version). This remains an interesting open problem, which we conjecture might also help find a complete characterisation of aggregation in the Markovian semantics.

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